MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2022/2023

PES0024 – ESSENTIAL STATISTICS

(All Sections/Groups)

06 JULY 2023
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of TWELVE (**12**) printed pages with:
   Section A: Ten (10) multiple choice questions (20 marks)
   Section B: Four (4) structured questions (80 marks)

2. Attempt **ALL** questions.

3. Students are allowed to use non-programmable scientific calculators (without restriction).

4. Please write all your answers in the Answer Booklet provided. All necessary workings **MUST** be shown.

5. Statistical formulas and tables are provided in the appendix.

SECTION A [20 marks]

1) What is the probability of impossible event?

a) 0
b) 1
c) 1/2
d) 1/4

2) What will be the probability of getting odd numbers if a dice is thrown?

a) 1/2
b) 2
c) 4/2
d) 5/2

3) Let X be the random variable, the probability distribution table is given by

X	0	1	2	3
P(X=x)	1/8	1/2	1/8	1/4

Find the value of $P(X \geq 1)$

a) 5/7
b) 7/8
c) 3/8
d) 8/9

4) For the random variable X with probability density function $f(x) = \frac{(x-3)^2}{5}$; x=3,4,5, the variance of X is:

a) $\frac{2}{25}$
b) $\frac{2}{5}$
c) $\frac{4}{5}$
d) $\frac{4}{25}$

5) If random variable X follows binomial distribution with parameter n and p are 8 and is 0.48 respectively, then the $P(X \leq 2)$ is:

a) 0.0053
b) 0.0448
c) 0.1724
d) 0.4078

Continued...

6) . Heights of college women have a distribution that can be approximated by a normal curve with a mean of 65 inches and a standard deviation equal to 3 inches. About what proportion of college women are between 65 and 67 inches tall?

a) 0.75
b) 0.50
c) 0.25
d) 0.17

7) Suppose that vehicle speeds at an interstate location have a normal distribution with a mean equal to 70 mph and standard deviation equal to 8 mph. What is the z-score for a speed of 64 mph?

a) −0.75
b) +0.75
c) −6
d) +6

8) Pulse rates of adult men are approximately normal with a mean of 70 and a standard deviation of 8. Which choice correctly describes how to find the proportion of men that have a pulse rate greater than 78?

a) Find the area to the left of $z = 1$ under a standard normal curve.
b) Find the area between $z = -1$ and $z = 1$ under a standard normal curve.
c) Find the area to the right of $z = 1$ under a standard normal curve.
d) Find the area to the right of $z = -1$ under a standard normal curve.

9) Which statement is not true about confidence intervals?

a) A confidence interval is an interval of values computed from sample data that is likely to include the true population value.
b) An approximate formula for a 95% confidence interval is sample estimate ± margin of error.
c) A confidence interval between 20% and 40% means that the population proportion lies between 20% and 40%.
d) A 99% confidence interval procedure has a higher probability of producing intervals that will include the population parameter than a 95% confidence interval procedure.

10) Suppose x is a Poisson-distributed random variable with an expected value of 5 occurrences per interval. What is p(x=3)?

a) 0.2004
b) 0.1404
c) 0.1704
d) 0.0904

Continued...

SECTION B
QUESTION 1 [20 marks]

a) Consider the word "SURVIVING." The individual letters of this word are written on separate cards, shuffled, and arranged in a line.

i) In how many ways can the cards be arranged? (2 marks)

ii) Three letters are chosen at random. Find the probability that at most two letters chosen are consonants. (4 marks)

b) Based on the records of NX Bank, 75% of its car loans have been fully repaid. Further analysis reveals that among the unpaid loans, 85% were granted to applicants employed at their current job for less than two years. Conversely, among the repaid loans, 25% were granted to applicants with employment durations of less than two years at their present jobs.

i) Draw a tree diagram for the above problem. (3 marks)

ii) What is the probability that a particular loan applicant would have been employed at his or her present job for less than two years? (2 marks)

iii) Given that a particular loan applicant has spent only one year on the present job, what is the probability that this person will repay the loan? (3 marks)

c) The provided table presents event data related to computer sales at a fictional store. The frequencies of sales are categorized based on the customer's gender and the type of computer purchased. The cell values indicate the number of PCs and Macs bought by both genders.

	PC	Mac	Total
Male	66	40	106
Female	30	97	127
Total	96	137	233

Suppose a customer is selected at random. Find the probability that the selected individual:

i) is a male or purchase PC. (3 marks)

ii) purchase Mac given that the individual is a female. (3 marks)

Continued...

QUESTION 2 [20 marks]

a) A small coffee shop located in MMU conducted a basic survey among students who frequently purchase coffee from their establishment. Let X the number of coffees purchased on a weekly basis. The data collected is presented in the table below:

x	1	2	3	4	5
$P(X = x)$	0.50	0.23	0.13	0.08	0.06

i) Validate the information above as a probability distribution of X. (2 marks)

ii) Calculate the mean and standard deviation of X. (8 marks)

iii) Find the probability that a randomly chosen a customer will purchase exactly 3 cups of coffee in a week? (1 mark)

iv) Find the probability that a customer will purchase at least 2 cups of coffee in a week? (2 marks)

b) The time in minutes taken for cars to find parking in Tamarind Square can be modelled by the continuous random variable with probability density function given by

$$f(x) = \begin{cases} k(6-x), & \text{for } 0 \leq x \leq 6 \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant.

i) Show that $k = 1/18$. (3 marks)

ii) Find the mean time taken for cars to find parking in Tamarind Square. (4 marks)

Continued...

QUESTION 3 [20 marks]

a) The probability that a certain type of vacuum tube will shatter during a thermal shock test is 0.15. If 25 such tubes are tested, find:

i) the probability that exactly 4 will shatter. (2 marks)

ii) the expected number of vacuum tube that will shatter during the test. (2 marks)

b) An online shop sells a computer game at an average rate of 4 per day. Find the probability that the shop sells:

i) exactly 20 in a given week. (3 marks)

ii) less than 4 units in a day. (3 marks)

c) In a recent study of baristas' hourly wages at Cyberjaya, it was found that the mean hourly salary was RM 10.50, with a standard deviation of RM 1.30. Assuming a normal distribution of hourly wages, calculate the probability of a randomly selected barista member earning:

i) between RM 10 and RM 12 per hour? (5 marks)

ii) more than RM 14 per hour? (3 marks)

iii) less than RM 6 per hour? (2 marks)

Continued...

QUESTION 4 [20 marks]

a) The provided data represents the years of working experience for all six staff members of a small company. The dataset is as follows:

5, 5, 10, 7, 12, 11

i) Let x denote the years of working experience for all six staff members of this department. Tabulate the population distribution of x.

(3 marks)

ii) List all the possible samples of size five, without replacement, that can be selected from this population. Then, calculate the mean for each of these samples.

(9 marks)

iii) Establish the sampling distribution of $\bar{x}$. (3 marks)

b) Based on a random sample of 100 records, the running times, in seconds, of children at a school for a 100m race follow a normal distribution with a mean of 16.12 and a standard deviation of 1.60.

i) What is the point estimate of the mean running times?

(1 mark)

ii) Construct a 97% confidence interval for the mean running times, in seconds, of children at school for a 100m race.

(4 marks)

End of questions.

APPENDIX

STATISTICAL FORMULA

A. DESCRIPTIVE STATISTICS

Ungrouped data

	Population	Sample
Mean :	$\mu = \frac{\sum X}{n}$	$\bar{X} = \frac{\sum x}{n}$
Variance :	$\sigma^2 = \frac{\sum (X-\mu)^2}{N}$	$s^2 = \frac{\sum (x-\bar{x})^2}{n-1}$
	or	or
	$\sigma^2 = \frac{\sum X^2}{N} - \mu^2$	$s^2 = \frac{\sum x^2}{n-1} - \frac{(\sum x)^2}{n(n-1)}$
Standard Deviation :	$\sigma = \sqrt{\sigma^2}$	$s = \sqrt{s^2}$

Grouped data

	Population	Sample
Mean :	$\mu = \frac{\sum fX}{N}$	$\bar{X} = \frac{\sum fx}{n}$
	where X is the observation value or the midpoint of the interval.	where x is the observation value or the midpoint of the interval.
Variance :	$\sigma^2 = \frac{\sum f(X-\mu)^2}{N}$	$s^2 = \frac{\sum f(x-\bar{x})^2}{n-1}$
	or	or
	$\sigma^2 = \frac{\sum fX^2}{N} - \mu^2$	$s^2 = \frac{\sum fx^2}{n-1} - \frac{(\sum fx)^2}{n(n-1)}$
	where X is the observation value or the midpoint of the interval.	where x is the observation value or the midpoint of the interval.

Continued...

Standard Deviation :	$\sigma = \sqrt{\sigma^2}$	$s = \sqrt{s^2}$

Other measures

- Median for the grouped data:

$$Median = L_m + \left[\frac{(n/2) - \sum f_{m-1}}{f_m}\right] \times C$$

where n = sample size,
L_m = lower limit of the median class,
$\sum f_{m-1}$ = cumulative frequency before the median class,
f_m = frequency of the median class, and
C = median class size.

- Mode for the grouped data:

$$Mode = L + \left[\frac{f_0 - f_1}{(f_0 - f_1) + (f_0 - f_2)}\right] \times C$$

where L = lower boundary of the class containing mode,
C = size of the class containing mode,
f_0 = frequency of the class containing mode,
f_1 = frequency of the class before the class containing mode, and
f_2 = frequency of the class after the class containing mode.

B. PROBABILITY

Basic Probability Rules

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) \cdot P(B)$ if A and B are independent

$P(A \mid B) = P(A \cap B) \div P(B)$

C. RANDOM VARIABLES

Discrete Random Variable

Mean: $E(X) = \sum xP(x)$

Variance: $\text{Var}(X) = E(X^2) - [E(X)]^2$ where $E(X^2) = \sum x^2 P(x)$

Continued...

Continuous Random Variable

Mean: $E(X) = \int_{-\infty}^{\infty} xf(x)\,dx$

Variance: $\text{Var}(X) = E(X^2) - [E(X)]^2$ where $E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

D. PROBABILITY DISTRIBUTIONS

Binomial Probability Distribution

$X \sim \text{B}(n, p)$

Probability: $P(X = x) = {}^nC_x p^x q^{n-x}$

Mean: $\mu = E(X) = np$

Variance: $\sigma^2 = Var(X) = npq$ where $q = 1 - p$

Poisson Probability Distribution

$X \sim \text{Po}(\lambda)$

Probability: $P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$

Mean: $\mu = E(X) = \lambda$

Variance: $\sigma^2 = Var(X) = \lambda$

Normal Probability Distribution

$X \sim \text{N}(\mu, \sigma)$ where the mean = $E(X) = \mu$ and variance = $Var(X) = \sigma^2$.

If X is standardized to $Z \sim \text{N}(0, 1)$, then $z = (x - \mu)/\sigma$.

E. SAMPLING DISTRIBUTIONS

Mean: $\mu_{\bar{x}} = \mu$ Standard deviation: $\sigma_{\bar{x}} = \sigma/\sqrt{n}$

F. CONFIDENCE INTERVAL

$(100 - \alpha)\%$ confidence interval for population mean: $\overline{X} \pm Z_{\alpha/2}\left(\dfrac{\sigma}{\sqrt{n}}\right)$

Margin of error: $Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$

Continued...

Confidence level:

Confidence level	$\alpha/2$	Z value
90%	5%	**1.645**
95%	2.5%	1.96
96%	2%	2.05
97%	1.5%	2.17
98%	1%	2.33
99%	0.5%	**2.58**

STATISTICAL TABLES

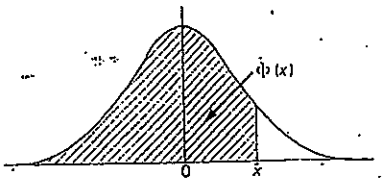
TABLE 2. THE POISSON DISTRIBUTION FUNCTION

μ	$r=0$	1	2	3	4	5	6	7	8	9	10
3·50	0·0302	0·1359	0·3208	0·5366	0·7254	0·8576	0·9347	0·9733	0·9901	0·9967	0·9990
·55	·0287	·1307	·3117	·5259	·7160	·8509	·9308	·9713	·9893	·9963	·9989
·60	·0273	·1257	·3027	·5152	·7064	·8441	·9267	·9692	·9883	·9960	·9987
·65	·0260	·1209	·2940	·5046	·6969	·8372	·9225	·9670	·9873	·9956	·9986
·70	·0247	·1162	·2854	·4942	·6872	·8301	·9182	·9648	·9863	·9952	·9984
3·75	0·0235	0·1117	0·2771	0·4838	0·6775	0·8229	0·9137	0·9624	0·9852	0·9947	0·9983
·80	·0224	·1074	·2689	·4735	·6678	·8156	·9091	·9599	·9840	·9942	·9981
·85	·0213	·1032	·2609	·4633	·6581	·8081	·9044	·9573	·9828	·9937	·9979
·90	·0202	·0992	·2531	·4532	·6484	·8006	·8995	·9546	·9815	·9931	·9977
·95	·0193	·0953	·2455	·4433	·6386	·7929	·8945	·9518	·9801	·9925	·9974
4·00	0·0183	0·0916	0·2381	0·4335	0·6288	0·7851	0·8893	0·9489	0·9786	0·9919	0·9972
·05	·0174	·0880	·2309	·4238	·6191	·7773	·8841	·9458	·9771	·9912	·9969
·10	·0166	·0845	·2238	·4142	·6093	·7693	·8786	·9427	·9755	·9905	·9966
·15	·0158	·0812	·2169	·4047	·5996	·7613	·8731	·9394	·9738	·9897	·9963
·20	·0150	·0780	·2102	·3954	·5898	·7531	·8675	·9361	·9721	·9889	·9959

Continued...

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\, dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.



x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0·00	0·5000	0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·97725
·01	·5040	·41	·6591	·81	·7910	·21	·8869	·61	·9463	·01	·97778
·02	·5080	·42	·6628	·82	·7939	·22	·8888	·62	·9474	·02	·97831
·03	·5120	·43	·6664	·83	·7967	·23	·8907	·63	·9484	·03	·97882
·04	·5160	·44	·6700	·84	·7995	·24	·8925	·64	·9495	·04	·97932
0·05	0·5199	0·45	0·6736	0·85	0·8023	1·25	0·8944	1·65	0·9505	2·05	0·97982
·06	·5239	·46	·6772	·86	·8051	·26	·8962	·66	·9515	·06	·98030
·07	·5279	·47	·6808	·87	·8078	·27	·8980	·67	·9525	·07	·98077
·08	·5319	·48	·6844	·88	·8106	·28	·8997	·68	·9535	·08	·98124
·09	·5359	·49	·6879	·89	·8133	·29	·9015	·69	·9545	·09	·98169
0·10	0·5398	0·50	0·6915	0·90	0·8159	1·30	0·9032	1·70	0·9554	2·10	0·98214
·11	·5438	·51	·6950	·91	·8186	·31	·9049	·71	·9564	·11	·98257
·12	·5478	·52	·6985	·92	·8212	·32	·9066	·72	·9573	·12	·98300
·13	·5517	·53	·7019	·93	·8238	·33	·9082	·73	·9582	·13	·98341
·14	·5557	·54	·7054	·94	·8264	·34	·9099	·74	·9591	·14	·98382
0·15	0·5596	0·55	0·7088	0·95	0·8289	1·35	0·9115	1·75	0·9599	2·15	0·98422
·16	·5636	·56	·7123	·96	·8315	·36	·9131	·76	·9608	·16	·98461
·17	·5675	·57	·7157	·97	·8340	·37	·9147	·77	·9616	·17	·98500
·18	·5714	·58	·7190	·98	·8365	·38	·9162	·78	·9625	·18	·98537
·19	·5753	·59	·7224	·99	·8389	·39	·9177	·79	·9633	·19	·98574
0·20	0·5793	0·60	0·7257	1·00	0·8413	1·40	0·9192	1·80	0·9641	2·20	0·98610
·21	·5832	·61	·7291	·01	·8438	·41	·9207	·81	·9649	·21	·98645
·22	·5871	·62	·7324	·02	·8461	·42	·9222	·82	·9656	·22	·98679
·23	·5910	·63	·7357	·03	·8485	·43	·9236	·83	·9664	·23	·98713
·24	·5948	·64	·7389	·04	·8508	·44	·9251	·84	·9671	·24	·98745
0·25	0·5987	0·65	0·7422	1·05	0·8531	1·45	0·9265	1·85	0·9678	2·25	0·98778
·26	·6026	·66	·7454	·06	·8554	·46	·9279	·86	·9686	·26	·98809
·27	·6064	·67	·7486	·07	·8577	·47	·9292	·87	·9693	·27	·98840
·28	·6103	·68	·7517	·08	·8599	·48	·9306	·88	·9699	·28	·98870
·29	·6141	·69	·7549	·09	·8621	·49	·9319	·89	·9706	·29	·98899
0·30	0·6179	0·70	0·7580	1·10	0·8643	1·50	0·9332	1·90	0·9713	2·30	0·98928
·31	·6217	·71	·7611	·11	·8665	·51	·9345	·91	·9719	·31	·98956
·32	·6255	·72	·7642	·12	·8686	·52	·9357	·92	·9726	·32	·98983
·33	·6293	·73	·7673	·13	·8708	·53	·9370	·93	·9732	·33	·99010
·34	·6331	·74	·7704	·14	·8729	·54	·9382	·94	·9738	·34	·99036
0·35	0·6368	0·75	0·7734	1·15	0·8749	1·55	0·9394	1·95	0·9744	2·35	0·99061
·36	·6406	·76	·7764	·16	·8770	·56	·9406	·96	·9750	·36	·99086
·37	·6443	·77	·7794	·17	·8790	·57	·9418	·97	·9756	·37	·99111
·38	·6480	·78	·7823	·18	·8810	·58	·9429	·98	·9761	·38	·99134
·39	·6517	·79	·7852	·19	·8830	·59	·9441	·99	·9767	·39	·99158
0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·9772	2·40	0·99180

Continued...

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2·40	0·99180	2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918
·41	·99202	·56	·99477	·71	·99664	·86	·99788	·01	·99869	·16	·99921
·42	·99224	·57	·99492	·72	·99674	·87	·99795	·02	·99874	·17	·99924
·43	·99245	·58	·99506	·73	·99683	·88	·99801	·03	·99878	·18	·99926
·44	·99266	·59	·99520	·74	·99693	·89	·99807	·04	·99882	·19	·99929
2·45	0·99286	2·60	0·99534	2·75	0·99702	2·90	0·99813	3·05	0·99886	3·20	0·99931
·46	·99305	·61	·99547	·76	·99711	·91	·99819	·06	·99889	·21	·99934
·47	·99324	·62	·99560	·77	·99720	·92	·99825	·07	·99893	·22	·99936
·48	·99343	·63	·99573	·78	·99728	·93	·99831	·08	·99896	·23	·99938
·49	·99361	·64	·99585	·79	·99736	·94	·99836	·09	·99900	·24	·99940
2·50	0·99379	2·65	0·99598	2·80	0·99744	2·95	0·99841	3·10	0·99903	3·25	0·99942
·51	·99396	·66	·99609	·81	·99752	·96	·99846	·11	·99906	·26	·99944
·52	·99413	·67	·99621	·82	·99760	·97	·99851	·12	·99910	·27	·99946
·53	·99430	·68	·99632	·83	·99767	·98	·99856	·13	·99913	·28	·99948
·54	·99446	·69	·99643	·84	·99774	·99	·99861	·14	·99916	·29	·99950
2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918	3·30	0·99952

The critical table below gives on the left the range of values of x for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

3·075	0·9990	3·263	0·9994	3·731	0·99990	3·916	0·99995
3·105	0·9991	3·320	0·9995	3·759	0·99991	3·976	0·99996
3·138	0·9992	3·389	0·9996	3·791	0·99992	4·055	0·99997
3·174	0·9993	3·480	0·9997	3·826	0·99993	4·173	0·99998
3·215	0·9994	3·615	0·9998	3·867	0·99994	4·417	0·99999
			0·9999		0·99995		1·00000

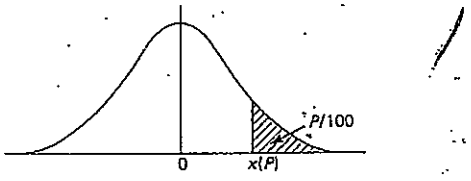
When $x > 3{\cdot}3$ the formula $1-\Phi(x) \doteq \frac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\frac{1}{x^2}+\frac{3}{x^4}-\frac{15}{x^6}+\frac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}}\int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geqslant x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geqslant x(P)$ is $2P/100$.



P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0·0000	5·0	1·6449	3·0	1·8808	2·0	2·0537	1·0	2·3263	0·10	3·0902
45	0·1257	4·8	1·6646	2·9	1·8957	1·9	2·0749	0·9	2·3656	0·09	3·1214
40	0·2533	4·6	1·6849	2·8	1·9110	1·8	2·0969	0·8	2·4089	0·08	3·1559
35	0·3853	4·4	1·7060	2·7	1·9268	1·7	2·1201	0·7	2·4573	0·07	3·1947
30	0·5244	4·2	1·7279	2·6	1·9431	1·6	2·1444	0·6	2·5121	0·06	3·2389
25	0·6745	4·0	1·7507	2·5	1·9600	1·5	2·1701	0·5	2·5758	0·05	3·2905
20	0·8416	3·8	1·7744	2·4	1·9774	1·4	2·1973	0·4	2·6521	0·01	3·7190
15	1·0364	3·6	1·7991	2·3	1·9954	1·3	2·2262	0·3	2·7478	0·005	3·8906
10	1·2816	3·4	1·8250	2·2	2·0141	1·2	2·2571	0·2	2·8782	0·001	4·2649
5	1·6449	3·2	1·8522	2·1	2·0335	1·1	2·2904	0·1	3·0902	0·0005	4·4172

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